

$\frac{15}{15}$

Quiz I: MTH 213, Spring 2018

Ayman Badawi

QUESTION 1. Find $gcd(44, 186)$. Then find $k_1, k_2 \in Z$ such that $44k_1 + 186k_2 = gcd(44, 186)$

$\frac{5}{5}$

$186 = 44 \times 4 + 10$ ($186 - 176 = 10$)

$44 = 10 \times 4 + 4$ ($44 - 40 = 4$)

$10 = 4 \times 2 + 2$ ($10 - 8 = 2$)

$4 = 2 \times 2 + 0$

$gcd(44, 186) = 2$ ✓

$2 = 10 - 4 \times 2$

$= 10 - (44 - 10 \times 4) \times 2$

$= 10 - (44 \times 2 - 10 \times 8)$

$= -44 \times 2 + 10 \times 9$

$= -2 \times 44 + (186 - 44 \times 4) \times 9$

$= -2 \times 44 + 186 \times 9 - 36 \times 44$

$= -38 \times 44 + 186 \times 9$

$\therefore k_1 = -38$ ✓

$k_2 = 9.$ ✓

QUESTION 2. Solve over Z, $6x = 9 \pmod{15}$

$6x = 9 \quad Z_{15}$

$gcd(6, 15) = 3$

$\therefore$ 3 solutions

$\frac{4}{4}$

$x = 4$ ✓

$x = 9$ ✓

$x = 14$ ✓

$\therefore$ Over Z

$x = 4 + 15a$ ✓

$x = 9 + 15b$ ✓

$x = 14 + 15c$ ✓

QUESTION 3. Find $(2341)_5 + (4434)_5$

$$\begin{array}{r} (2341)_5 \\ +(4434)_5 \\ \hline (12330)_5 \end{array}$$

$\frac{1}{1}$

QUESTION 4. Let x be number of defective mobiles in a particular store. Given,

$$\underset{r_1}{x \equiv 4} \ (mod\ \underset{m_1}{8}),\ \underset{r_2}{x \equiv 2} \ (mod\ \underset{m_2}{11}),\ \underset{r_3}{x \equiv 4} \ (mod\ \underset{m_3}{9})$$

. If $0 < x < 792$, find x

– gcd between m_i's is 1.

① $(99)^{-1} \bmod 8 = 3$

② $(72)^{-1} \bmod 11 = 2$

③ $(88)^{-1} \bmod 9 = 4$

$\frac{5}{5}$

for ① $99x = 1 \in Z_8$

$x = 3$.

for ② $72x = 1 \in Z_{11}$

$x = 2$

for ③ $88x = 1 \in Z_9$

$x = 4$

$\therefore x = [99 \times 3 \times 4 + 72 \times 2 \times 2 + 88 \times 4 \times 4] \bmod 792$

$= [2884] \bmod 792$

$x = 508$

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Quiz II: MTH 213, Spring 2018

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QUESTION 1. Let x be a rational number and y be an integer. Prove that $x+y$ is a rational number.

proof : let $x = \frac{x_1}{x_2}$ where $x_1 \in Z$ and $x_2 \in Z^*$

let $y = \frac{y_1}{y_2}$ where $y_1 \in Z$ and $y_2 \in Z^*$

Adding them: $x+y = \frac{x_1}{x_2} + \frac{y_1}{y_2}$ $y = \frac{x_1 + y.x_2}{x_2}$

Assuming $x_2 \neq y_2$,

$\frac{x_1 y_2 + y_1 x_2}{x_2 y_2}$

let $x_1 y_2 + y_1 x_2 = c \in Z$ Hence $x+y = \frac{c}{d} \in Q$.

let $x_2 y_2 = d \in Z^*$

4/5

QUESTION 2. Let x be an even integer an y be an odd integer. Prove that $x+y$ is an odd number. (i.e., show that $x+y = 2m+1$ for some integer $m \in Z$)

let $x = 2n_1$ and $y = 2n_2 + 1$, where $n_1, n_2 \in Z$

Adding : $x+y = 2n_1 + 2n_2 + 1$

$= 2(n_1 + n_2) + 1$

let $n_1 + n_2 = m \in Z$ (integer + integer)

Hence $x+y = 2m+1$ → odd number.

✓ 5/5

QUESTION 3. Convince me that there are integers $x, y \in Z$ such that $6x + 9y = 60$ (do not find the values of x and y)

gcd $(6,9) = 3$ where $c_1, c_2 \in Z$

which can be written as $3 = 6c_1 + 9c_2$, since $3 \mid 60$;

multiplying by 20 gives us

$3 \times 20 = 6(20)c_1 + 9(20)c_2$

let $20c_1 = x \in Z$ and $20c_2 = y \in Z$ (integer × integer)

Hence $60 = 6x + 9y$, x & y exist.

✓ 5/5

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Quiz III: MTH 213, Spring 2018

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QUESTION 1. Prove that $\sqrt{35}$ is irrational (Use contradiction)

Deny. Say $\sqrt{35}$ is rational.

$\sqrt{35} = \frac{a}{b}$, $a, b \in \mathbb{Z}$, $b \neq 0$, $\gcd(a,b) = 1$. and a and b are odd numbers.

let $a = 2m+1$ and $b = 2k+1$

$\Rightarrow \sqrt{35} = \frac{2m+1}{2k+1}$.

Square $\Rightarrow$ $35(4k^2+4k+1) = 4m^2+4m+1$

$140k^2 + 140k + 34 = 4m^2 + 4m$.

Divide by 4 $\Rightarrow$ $35k^2 + 35k + \frac{34}{4} = m^2 + m$.

8/8

$4 \nmid 34$

but LHS is not integer and RHS is integer (Contradiction) Hence $\sqrt{35}$ is irrational.

QUESTION 2. Prove that $(\sqrt{7} + \sqrt{5})$ is irrational (Use contradiction, and use (1), note that $\sqrt{35} = \sqrt{5}\sqrt{7}$)

Deny. Say $\sqrt{7} + \sqrt{5}$ ~~is~~ is rational.

i.e $\sqrt{7} + \sqrt{5} = \frac{a}{b}$, $\sqrt{7}$ & $\sqrt{5}$ are irrational, $a \in \mathbb{Z}$, $b \in \mathbb{Z}^*$

~~Multiply.~~ ~~Rearrange $\Rightarrow$ $\sqrt{7} = \frac{a}{b} - \sqrt{5}$~~

~~Multiply b with $\sqrt{5}$ $\Rightarrow$ $\sqrt{5}\sqrt{7} = \sqrt{5}\frac{a}{b} - \sqrt{5}\sqrt{5}$~~

~~$\sqrt{35} = \sqrt{5}\frac{a}{b} - 5$~~

Square both sides:

$(\sqrt{7} + \sqrt{5})^2 = \frac{a}{b}$

$7 + 2\sqrt{5}\sqrt{7} + 5 = \frac{a}{b}$

$2\sqrt{35} = \frac{a}{b} - 12$

$\sqrt{35} = \left(\frac{a}{b} - 12\right) \div 2$,

$= \frac{a-12b}{2b}$

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let $a - 12b = c$, $c \in \mathbb{Z}$

and $2b = d$ $d \in \mathbb{Z}$

$\therefore \sqrt{35} = \frac{c}{d}$

7/7

$\sqrt{35}$ is irrational, as shown in Q1, but $\frac{c}{d}$ is ~~ir~~ rational

Irrational $\neq$ Rational (contradiction)

Hence, $\sqrt{7} + \sqrt{5}$ is irrational.

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Quiz IV: MTH 213, Spring 2018

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QUESTION 1. Use Truth table (it is not matter, use T (or 1) and F (or 0)) to convince me, without any doubt, that $\neg(S_1 \Rightarrow S_2) \equiv (S_1 \wedge \neg S_2)$

S_1	S_2	$-S_1$	$-S_2$	$S_1 \Rightarrow S_2$	$-(S_1 \Rightarrow S_2)$	$S_1 \cap -S_2$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	F	F

7/7

$\therefore -(S_1 \Rightarrow S_2) \equiv (S_1 \cap -S_2)$

QUESTION 2. Let $F = \{2, 5, \{7\}, \{2\}, 7, x, \{x, 2\}, \phi\}$ and $D = \{5, \{7\}, \{x, 2\}\}$.
Write down F or T $\Rightarrow$ $F \cup T = \{2, 5, \{7\}, \{2\}, \{x, 2\}, x, \emptyset, 7\}$

(i) $\{7\} \subset F$ **T**

(ii) $\{\{x, 2\}, 7\} \subset F$ **T**

(iii) $\{7\} \subset F \cap D$ **F**

(iv) $\phi \in F \cup D$ **T**

(v) $\{2, 5\} \in P(F)$ (note $P(F)$ is the power set of F) **T**

(vi) $\{\phi, \{2\}\} \subset F$ **T**

(vii) Find $F - D$ $F - D = \{2, \{2\}, 7, x, \phi\}$

6/6

$P(F) = \{\emptyset, F, \{2\}, \{5\}, \ldots\}$

2/2

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Quiz 5: MTH 213, Spring 2018

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QUESTION 1. (i)

(ii) Let $f: \{1,2,3,4,5,6\} \to \{1,2,3,4,5,6\}$ such that $f(1)=2$, $f(2)=3$, $f(3)=1$, $f(4)=5$, $f(5)=6$, $f(6)=4$. i.e., $f == \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 6 & 4 \end{pmatrix}$. Find f^2 and f^3. Write f as a composition of disjoint cycles (as in class), then find the smallest positive integer $n \geq 1$ such that $f^n = I$.

ans: f as composition of disjoint cycles : $(1\ 2\ 3)(4\ 5\ 6)$ 1/1

$f^2 = (3\ 1\ 2\ 6\ 4\ 5)$ ← Range of f^2 1.5

$f^3 = (1\ 2\ 3\ 4\ 5\ 6)$ ← Range of f^3 1.5

$n = LCM(3,3) = 3$ 1

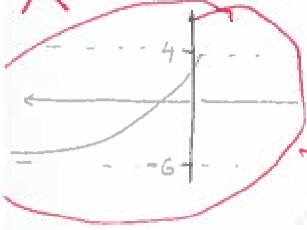
So $f^3 = I$ as shown above

5/5

(iii) Convince me that $|(-\infty, 0)| = |(-6, 4]|$

OK OK

Need a bijective function s.t $f: (-\infty, 0) \to (-6, 4]$



$f(x) = 10e^x - 6$

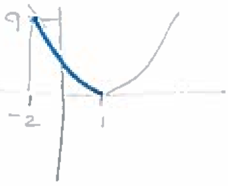
so $|(-\infty, 0]| = |(-6, 4]|$

now $(-\infty, 0] = (-\infty, 0) \cup \{0\}$

so $|(-\infty, 0]| = |(-\infty, 0)|$ and $|(-\infty, 0]| = |(-6, 4]|$

Hence $|(-\infty, 0)| = |(-6, 4]|$

(iv) Find a, b so that the function $f: (-2, a) \to (0, b)$, where $f(x) = (x-1)^2$ is bijective



$a = 1$ and $b = 9$

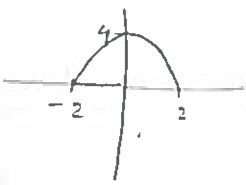
so $(-2, 1) \to (0, 9)$

(v) Let $F = Q \cap (3, 3.002)$. Is F countable or uncountable ? What is $|F|$?

F is a subset of $\mathbb{Q}$ and $\mathbb{Q}$ is countable, so F is also countable. Since there can be infinitely many rational number elements betw. two elements in F, $|F| = \infty = |\mathbb{Q}| = |\mathbb{N}|$

2/2

(vi) Let $f: (-2, 2) \to (0, 4)$, $f(x) = 4 - x^2$. Is $f(x)$ a function? if no, then can you make a small modification on the codomain so that $f(x)$ becomes a function that is onto?



$f(x)$ is not a function. To make $f(x)$ a function onto, codomain should be $(0, 4]$

because 0 in the domain (included) cannot be mapped to anything in the co-domain. (4 not included) (0 has no image)

2/2

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Quiz 5: MTH 213, Spring 2018

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1.5/5

QUESTION 1. (i)

(ii) Let $f : \{1,2,3,4,5,6\} \to \{1,2,3,4,5,6\}$ such that $f(1) = 2$, $f(2) = 3$, $f(3) = 1$, $f(4) = 5$, $f(5) = 6$, $f(6) = 4$. i.e., $f == \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 6 & 4 \end{pmatrix}$. Find f^2 and f^3. Write f as a composition of disjoint cycles (as in class), then find the smallest positive integer $n \geq 1$ such that $f^n = I$.

ans: f as composition of disjoint cycles: $(1\ 2\ 3)(4\ 5\ 6)$ 1/1

$f^2 = (3\ 1\ 2\ 6\ 4\ 5) \leftarrow$ Range of f^2 1.5/1.5

$f^3 = (1\ 2\ 3\ 4\ 5\ 6) \leftarrow$ Range of f^3 1.5/1.5

$n = LCM(3,3) = 3$ 1

So $f^3 = I$ as shown above

5/5

(iii) Convince me that $|(-\infty, 0)| = |(-6, 4]|$

OK OK

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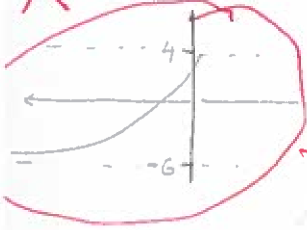
$f(x) = 10e^x - 6$



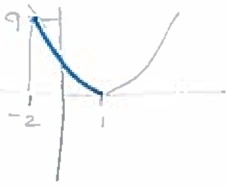
so $|(-\infty, 0]| = |[-6,4]|$

now $(-\infty, 0] = (-\infty, 0) \cup \{0\}$

so $|(-\infty, 0]| = |(-\infty, 0)|$ and $|(-\infty, 0]| = |(-6,4]|$

Hence $|(-\infty, 0)| = |(-6,4]|$

(iv) Find a, b so that the function $f : (-2, a) \to (0, b)$, where $f(x) = (x-1)^2$ is bijective



$a = 1$ and $b = 9$

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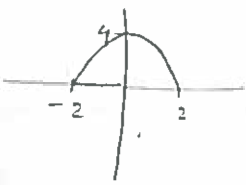
2/2

(vi) Let $f : (-2, 2) \to (0, 4)$, $f(x) = 4 - x^2$. Is $f(x)$ a function? if no , then can you make a small modification on the codomain so that $f(x)$ becomes a function that is onto?



$f(x)$ is not a function. To make $f(x)$ a function onto, codomain should be $[0, 4]$ (included) because 0 in the domain cannot be mapped to anything in the co-domain. (4 not included) (0 has no image)

2/2

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Quiz 6 & 7: MTH 213, Spring 2018

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$\frac{15}{15}$

QUESTION 1. Let $A = Z$. Define "=" on A so that for every $a, b \in A$, we have $a = b$ if $6 \mid (a - b)$ (in A). We know that = is an equivalence relation on A (do not check). Find all equivalence classes of A. For each equivalence class, describe all its elements.

$\frac{3}{3}$

Ans: • $[0] = \{\ldots, -12, -6, 0, 6, 12, 18, \ldots\}$ set of all elements "equal to" 0. For $6 \mid a - b$, $a \bmod 6 = b \bmod 6$. In this case, the element mod 6 = 0. i.e for every $a \in [0]$, $a = 0$, i.e $6 \mid a$, i.e $a \bmod 6 = 0$

• $[1] = \{\ldots, -11, -5, 1, 7, 13, 19, \ldots\}$ set of all elements "equal" to 1 i.e $\forall$ element $\in [1]$ element mod 6 = 1.

• $[2] = \{\ldots, -10, -4, 2, 8, 14, 20, \ldots\}$ set of all elements "equal" to 2 i.e $\forall a \in [2]$, $a \bmod 6 = 2$

• $[3] = \{\ldots, -9, -3, 3, 9, 15, 21, \ldots\}$ set of all elements "equal" to 3 i.e $\forall a \in [3]$, $a \bmod 6 = 3$

• $[4] = \{\ldots, -8, -2, 4, 10, 16, 22, \ldots\}$ set of all elements "equal" to 4. $\forall a \in [4]$, $a \bmod 6 = 4$

• $[5] = \{\ldots, -7, -1, 5, 11, 17, 23, \ldots\}$ set of all elements "equal" to 5. $\forall a \in [5]$, $a \bmod 6 = 5$

QUESTION 2. Let $A = \{-5, 2, 5, 9, 11, 17, 21\}$. Define = on A so that for every $a, b \in A$, we have $a = b$ if $b = ca$ for some $c \in \{1, -1, 3, -3\}$. Convince me that = is an equivalence relation on A. Find all equivalence classes. If we view = as a subset of $A \times A$, how many elements does = have?

$\frac{3}{3}$

Ans: check: finite set

A — A. Let $a \in A$. Show "$a = a$" i.e $a = c \cdot a$.
This axiom holds because for every element a in A, $a = a \cdot 1$ and $1 \in \{1, -1, 3, -3\}$.

A — B. Let $a, b \in A$. Assume "$a = b$". Show "$b = a$".
let $a = 5$ and $b = -5$. "$a = b$" means $-5 = 5 \cdot c \Rightarrow -5 = 5 \times -1$, and $-1 \in \{1, -1, 3, -3\}$.
~~x=-1~~ $\underset{a}{5} = \underset{b}{-5} \cdot (+c) \Rightarrow$ ~~5=-5(-1)~~
$5 = -5 \times (-1)$ and $-1 \in \{1, -1, 3, -3\}$.
Hence "$b = a$".

A – B – C. let $a, b, c \in A$. Assume "$a = b$" and "$b = c$". Show that "$a = c$".
There are no 3 distinct elements in A for statement 1 ("$a = b$" and "$b = c$") to be true. Hence by default, statement 2 ("$a = c$") is true.
$\therefore$ "=" is an equivalence relation on A.

Equivalence classes:

$[5] = \{5, -5\}$
$[2] = \{2\}$
$[9] = \{9\}$
$[11] = \{11\}$
$[17] = \{17\}$
$[21] = \{21\}$

$\frac{2}{2}$

No. of elements in "=" $= 2^2 + 1 + 1 + 1 + 1 + 1$
$= \underline{9}$

$P(A) = \{A, \phi, \{1\}, \{2\}, \{3\}, ...\}$

QUESTION 3. Let $A = \{1,2,3\}$. Define $\leq$ on $P(A)$ such that for every $a, b \in P(A)$, we have $a \leq b$ if $a \subseteq b$. We know that that $\leq$ is a partial order relation on $P(A)$ (note that $|P(A)| = 8$), do not show that. If possible find

(i) $\{2\} \wedge \{3\}$ $c \leq \{2\}$ and $c \leq \{3\}$

$\{2\} \wedge \{3\} = \phi$

(ii) $\{1\} \vee \{2\}$ $\{1\} \subseteq d$ and $\{2\} \subseteq d$

$\{1\} \vee \{2\} = \{1, 2\}$

(iii) $\{3\} \wedge \{1,3\}$ $c \leq \{3\}$ and $c \leq \{1,3\}$

$\{3\} \wedge \{1,3\} = \{3\}$

(iv) $\{2,3\} \wedge \{1,2\}$ $c \subseteq \{2,3\}$ and $c \subseteq \{1,2\}$

$\{2,3\} \wedge \{1,2\} = \{2\}$

(v) $\{1,3\} \vee \{2,3\}$ $\{1,3\} \subseteq d$ and $\{2,3\} \subseteq d$

~~$\{1,3\} \wedge$~~ $\{1,3\} \vee \{2,3\} = A = \{1,2,3\}$

2.5 / 2.5

(vi) I claim there is an element $d \in P(A)$ such that $d \leq f$ for every $f \in P(A)$. Find d. Also, I claim there is an element $m \in P(A)$ such that $a \leq m$ for every $a \in P(A)$. Find m.

$d \leq f$ means $d \subseteq f$ for every $f \in P(A)$.

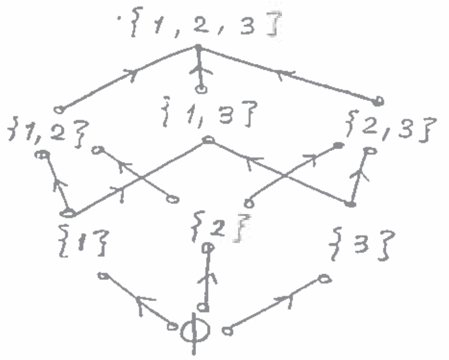
$d = \phi$

$a \leq m$ means $a \subseteq m$ for every $a \in P(A)$

~~$a = A$~~ $m = A = \{1, 2, 3\}$ (whole set)

2 / 2

(vii) Draw the Hasse diagram of $(P(A), \leq)$. [Hint: put ϕ down. Above ϕ put all sets with 1 element (line up)...above that put ...., then connect).



2.5 / 2.5

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Quiz 8: MTH 213, Spring 2018

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15/15

QUESTION 1. (i) $O(2x^3 + 7x - 3)$ = x^3 ~~1.5~~ 2 (each)

(ii) $O(\sqrt[3]{x^2} + \sqrt{x} + 2x - 4)$ = x ~~1.5~~ 2 (each)

QUESTION 2. Let $\{a_n\}_{n=0}^{\infty}$, such that $a_n = -6a_{n-1} - 9a_{n-2} + 2n + 10$, where $a_0 = 12$ and $a_1 = 3$. Find a general formula for a_n. Then find a_6.

i) let $b_n = -6b_{n-1} - 9b_{n-2}$

let $r^n = b_n$

$\Rightarrow \frac{r^n}{r^{n-2}} = \frac{-6r^{n-1}}{r^{n-2}} - \frac{9r^{n-2}}{r^{n-2}}$

$r^2 = -6r - 9$

$r^2 + 6r + 9 = 0$

$(r+3)(r+3) = 0$

4/4

$\Rightarrow r = -3$ (repeating)

$a_n = c_1(-3)^n + c_2 n(-3)^n + 2n + 10$ 2/2

$a_0 = c_1 + 10$ $\qquad$ $a_1 = c_1(-3) + c_2(-3)^1 + 2 + 10$

$c_1 = 12 - 10$ $\qquad$ $3 = -3c_1 - 3c_2 + 12$

$c_1 = 2$ $\qquad$ $3(2) + 3c_2 = 9$

$c_2 = 1$ 3/3

$a_n = 2(-3)^n + 1n(-3)^n + 2n + 10$ 1

ii) $a_6 = 2(-3)^6 + (6)(-3)^6 + 2(6) + 10$

$= 5854$ 1/1

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